Data Analysis 1: Scaling Relations (75 points)

Please read the general instructions in the separate envelope before you start this problem.

Spiral galaxies are disk-like rotating structures, whose dynamical state is fairly grasped by the so-called rotation curves, quantifying the mean rotational velocity of the disk at different distances from the center (see Figure 1, curve B). An interesting feature is the flat region of the curve, which is attributed to the presence of dark matter. Without it, rotation velocities would drop steadily at large radii from the center, as depicted in curve A.
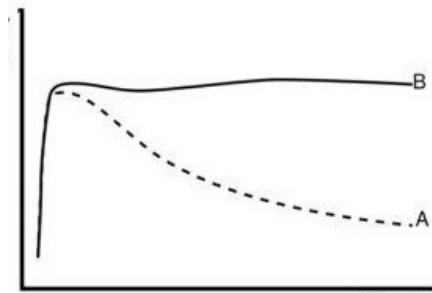

Figure 1: Rotation curves. Circular velocity (Y-axis) vs Radius (X-axis)

In disk galaxies a strong correlation has been observed between the intrinsic luminosity of the whole galaxy and the asymptotic rotational velocity (as given by the rotation curve for the outer edge of the galaxy i.e. R_{max}), a result that is known as the Tully-Fisher relation. This relation also holds if you use the luminosity in a specific band. This is shown on Figure 2 for a number of galaxies in a galaxy cluster. Every dot is a galaxy, and the solid line is the best-fit linear relation between absolute magnitude in K band and $log_{10}(V_{\mathrm{max}})$ for the whole sample.
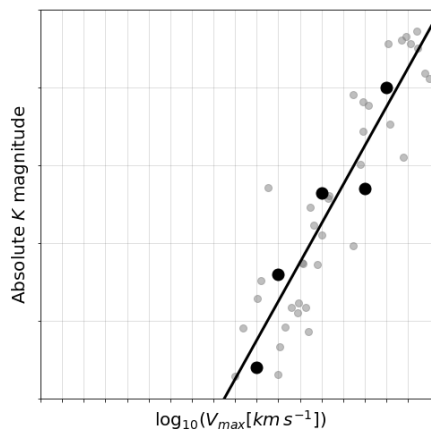

Figure 2: Absolute magnitude in K band vs $\log_{10}(V_{\mathrm{max}}[km\,s^{-1}])$. Tully-Fisher relation for several galaxies. Every dot represents a galaxy. The dark points are five selected galaxies, for which we will provide some numbers in part 1.2.
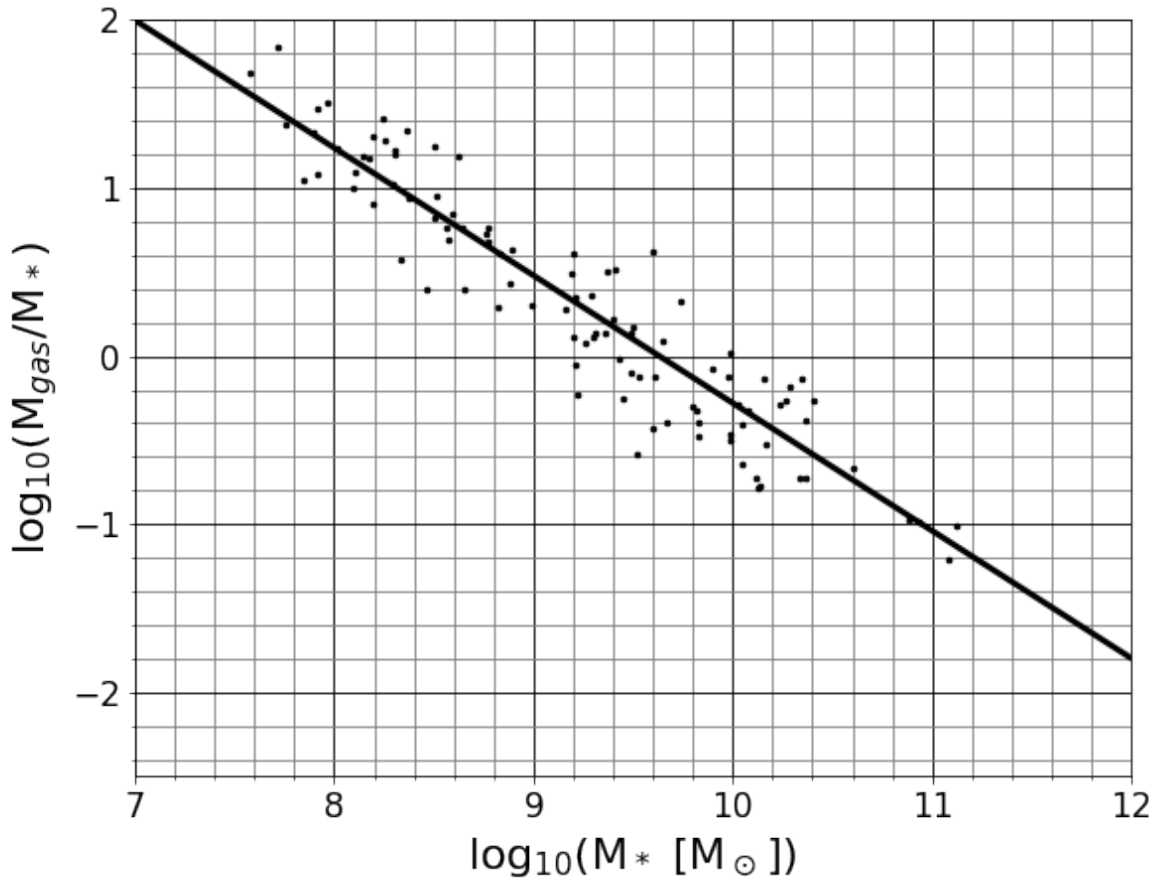
Figure 3: Gas fraction vs stellar mass.

Another interesting trend is shown in Figure 3: disks with larger stellar masses (M_*) tend to have smaller gas fractions (M_{gas}/M_*).

In the following questions you will be asked to extract physical information about the galaxies using the scaling relations just introduced. Consider the following guidelines:

- Assume that $V_{\max}$ was measured at the same radius for all galaxies ($R_{\max}$), in the flat part of the rotation curves and well beyond the end of the stellar disk.

- Use M_{dm} for the dark matter mass up to R_{max} and M_{tot} for the sum of all components.(gas, stars and dark matter)

- Assume that all galaxies have identical stellar populations[1], and assume that the gaseous component does not interact with the stellar light. .

- The galaxy cluster is far away. Its distance is much larger than the cluster size.

- In spherically-symmetric mass distributions, to infer the gravitational effect on a particle at distance r from the center, it suffices to consider the total mass enclosed up to that radius $M(\leq r)$ as if it were placed at the very center of the distribution.

[1]The term stellar population refers to the type of stars that are present in a galaxy, and the relative amount of each type with respect to the total number of stars.

Part 1 (20 points).

| 1.1 | From an analysis of Figure 3, find the appropriate constants in the following relation: $M_{gas} = a \times M_*^b$ | 5.0pt |

$$a = ?$$

$$b = ?$$

| 1.2 | In the plot of the Tully-Fisher relation there are 5 highlighted points. Data for these 5 galaxies is given in the following table. Use this dataset to find the appropriate constants for TF relation presented below the table, by means of a linear fit using the method of least squares. | 15.0pt |

Note: Treat $log_{10}(V_{\max})$ as the x variable and K as the y variable in the linear fit.

$V_{\max}[km/s]$	$K[mag]$
79.4	-16.8
100.1	-19.2
158.5	-21.3
251.2	-21.4
316.2	-24.0

$$K = c \times log_{10}(V_{max}) + d$$

$$c = ?$$

$$d = ?$$

Part 2 (16 points).

For two galaxies, G1 and G2, in the cluster, the recorded $apparent$ magnitudes are:

$$k_1 = 19.2 \qquad ; \qquad k_2 = 25.2$$

Using this information and the relations calibrated in Part 1 find the correct exponents in the following equations:

| 2.1 | | 6.0pt |

$$\frac{M_{*1}}{M_{*2}} = 10^e \qquad ; \qquad e = ?$$

2.2		4.0pt
	$$\frac{M_{gas1}}{M_{gas2}} = 10^f \quad ; \quad f = ?$$	

2.3		6.0pt
	$$\frac{M_{tot1}}{M_{tot2}} = 10^g \quad ; \quad g = ?$$	

Part 3 (15 points).

3.1		15.0pt

Galaxy	Apparent magnitude k	$M_{gas}[M_\odot]$	$M_* [M_\odot]$	$M_{dm}[M_\odot]$	$M_{tot} [M_\odot]$
G_1	19.2				4.39×10^{11}

Fill in the missing values in the table using the fact that for galaxy G_1, the dark-to-baryonic mass ratio up to $R_{\max}$ is 6.82.

Part 4 (24 points).

4.1	Consider a systematic uncertainty of $\sigma_{sys} = \pm 0.2$ in each apparent magnitude due to CCD calibration errors. Then k_1 must be read as $k_1 = 19.2 \pm 0.2$, i.e., the only thing we know is that k_1 most likely lies in the interval $[19.0, 19.4]$. The same goes for k_2. Recalculate the exponent in the scaling relation $\frac{M_{*1}}{M_{*2}} = 10^e$ (found in 2.1), expressing e as an interval estimated by considering the extreme possible variations in k_1 and k_2 .	4.0pt

$$e \in [?, ?]$$

4.2 Now we consider that there is always a natural spread of the data around any relation. For instance, for a given value of the K magnitude the TF relation gives a single value of $log_{10}(V_{\mathsf{max}})$, but it would be more realistic to report an interval of plausible values, derived from the natural spread of the data around the mean TF relation. We call this the statistical uncertainty, σ_{stat}.

Estimate the statistical uncertainty if $log_{10}(V_{\mathsf{max}})$ is inferred from K using the TF relation from question 1.2. For this, consider for each point the difference between the value of $log_{10}(V_{\mathsf{max}})$ estimated from K using your linear fit and the actual measurement of $log_{10}(V_{\mathsf{max}})$, and take σ_{stat} as two times the root mean square (RMS) of these differences[†].

10.0pt

$$\sigma_{stat} = ?$$

[†]The RMS of a set of values is the square root of the arithmetic mean of the squares of those values.

4.3 Recalculate the exponent in the scaling relation $\frac{M_{tot1}}{M_{tot2}} = 10^g$, expressing g as an interval estimated by considering the extreme possible variations arising from both the systematic and statistical uncertainties:

10.0pt

$$g \in [?, ?]$$

Data Analysis 2: Stars and Exoplanets (75 points)

Please read the general instructions in the separate envelope before you start this problem.

In this problem, we will explore the connection between the physical properties of exoplanets and their host stars and will use the observational data to discover as much as possible about these systems. You may neglect interstellar extinction.

Part 1 (20 points).

Name of planet	Name of star	$T_{eff}\,(K)$	$g\,(ms^{-2})$	m_v (magnitudes)	parallax (milliarcsec)
$Gorgona$	$HD\,209458$	5980	347	7.63	20.67

Table 1: Observational data for exoplanet Gorgona and its parent star *HD 209458*

The effective temperature (T_{eff}) and the gravitational acceleration in the surface of the star (g) can be measured from the shape of the spectrum and its absorption lines. The visual apparent magnitude (m_v) and parallax are measured by doing photometry and astrometry, respectively.

Additionally, it has been observed that every 3.52 days the brightness of the star drops due to the transit of the planet in front of it, as it is represented in this lightcurve:
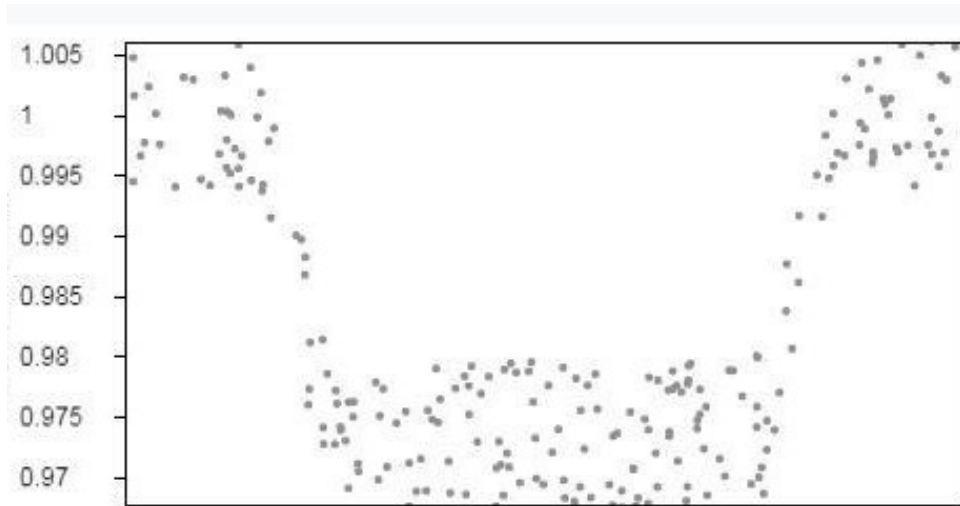
1.1 20.0pt

Figure 1: Normalized Flux (y-axis) against time (x-axis) for the parent star *HD 209458*

Use the given information to calculate the following quantities for the *HD 209458* system:

Luminosity of the star	Radius of the star	Mass of the star	Mean planet's orbital radius	Radius of the planet in Jupiter's radius
$L_\star$ $[L_\odot]$	$R_\star$ $[R_\odot]$	$M_\star$ $[M_\odot]$	a $[au]$	R_p $[R_J]$

Note: Assume that the bolometric correction for all F and G type stars is the same.

Part 2 (25 points).

The habitable zone is defined as the region in which a planet may have liquid water on its surface. This is mainly related to the amount of radiation received from the host star, which must be within some limits to ensure a favorable range of planet surface temperatures.
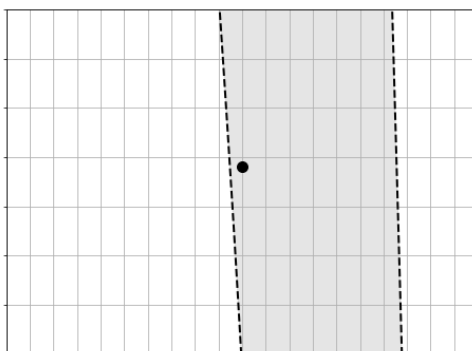
We define the effective flux received by a planet as $S_{eff} = \frac{L}{a^2}$, where L is the star luminosity in solar units, and a is the mean orbital radius in au. The minimum flux in the habitable zone can be approximated by $S_{min} = S_{eff_\odot} + n \cdot T_\star + b \cdot T_\star^2 + c \cdot T_\star^3 + d \cdot T_\star^4$, where $T_\star = (T_{eff} - T_{eff_\odot})$, and $S_{eff_\odot}$ is the equivalent flux for the case of the Sun, which along with the coefficients n, b, c, d is given in the following table. The maximum flux for habitability, S_{max}, is found with the same equation but different constants:

Constant	S_{max}	S_{min}
$S_{eff_\odot}$	1.0512	0.3438
n	1.3242×10^{-4}	5.8942×10^{-5}
b	1.5418×10^{-8}	1.6558×10^{-9}
c	-7.9895×10^{-12}	-3.0045×10^{-12}
d	-1.8328×10^{-15}	-5.2983×10^{-16}

The table below gives real data for 7 different star-planet systems. However planet names have been changed to honour some natural sanctuaries in Colombia:

Stellar Parameters		Planet Parameters	
$T_{eff}\,[K]$	$M_V\,[mag]$	Name	$a[au]$
6180	3.68	Tayrona	0.04
5730	3.87	Iguaque	0.04
5980	4.21	Gorgona	0.04
5480	6.04	Amacayacu	0.08
5770	3.48	Malpelo	0.05
6130	3.07	Pisba	0.03
6140	3.85	Tatamá	0.06

2.1 In the following figure, the vertical axis represents the effective temperature of stars, and the horizontal axis represents the effective flux received by orbiting planets. The dot marked on the graph represents planet Earth, and dashed lines mark the limits of the habitable zone.

15.0pt

Put numerical labels at tickmark position on both axes. Draw on the same figure the exact position where Gorgona and Amacayacu would be, as if they were also located at 1 au from their corresponding stars.

2.2 Now considering the real orbital radius given in the table for each planet, indicate with YES or NO which of them are in the habitable zone. Show your quantitative reasoning on the working sheets.

10.0pt

Planet's Name	In habitable zone? YES / NO
Tayrona	
Iguaque	
Gorgona	
Amacayacu	
Malpelo	
Pisba	
Tatamá	

Part 3 (30 points).

In the last page you find a list of 38 exoplanets, and the goal is to find out if low-mass exoplanets (LME) and high-mass exoplanets (HME) tend to orbit around stars with different characteristics.

3.1 To get a robust low-mass subsample one can apply a technique called "iterative sigma-clipping". The idea is to compute the mean (μ) and the standard deviation (σ) of the masses and to exclude from the sample, those planets with masses above $\mu + \sigma$. Then repeat the same steps with the remaining subsample two more times. We will say that planets in the final subsample are the low-mass ones, and those excluded during the iterations, the high-mass ones. Fill the following table with the numbers you find in the process: **10.0pt**
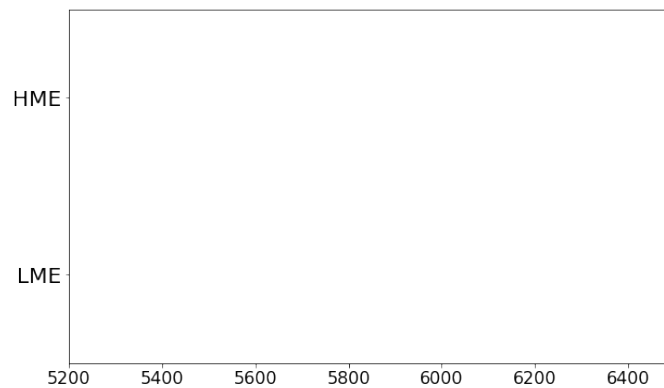
Low-mass sample	Sample size	μ	σ	$\mu + \sigma$	No. of planets to exclude
Full / Original	38				
subsample after 1st iteration					
subsample after 2nd iteration					
subsample after final iteration		---	---	---	---

3.2 Make a plot using the X-axis for the serial number of the planets in the list $(1, 2, 3, ...)$, and the Y-axis for the mass of the planets. Mark 3 horizontal lines at the $\mu + \sigma$ thresholds you found in the iterations: **5.0pt**

3.3 Let's investigate the possible difference in the effective temperatures of host stars in both groups, computing some descriptive statistics: **10.0pt**

	Min.	1st Quartile	Median 2nd quartile	3rd Quartile	Max
LME					
HME					

3.4 Draw boxplots summarizing the numbers you just computed. Do you see a clear difference in the temperatures of the stars orbited by low-mass and high-mass planets? Write YES or NO.

5.0pt

HME

LME

5200   5400   5600   5800   6000   6200   6400

Planetary type vs host star temperature (K)

No.	Name of Planet	Planet Mass $[M_J]$	T_{eff} of the Host Star
1	KEPLER-37 b	0.01	5520
2	KEPLER-21 b	0.02	6256
3	HD 97658 b	0.02	5468
4	HD 46375 b	0.23	5345
5	HD 219134 h	0.28	5209
6	HD 88133 b	0.30	5582
7	HD33283 b	0.33	5877
8	HD 149026 b	0.36	6096
9	BD-10 3166 b	0.46	5578
10	HD 75289 b	0.47	6196
11	HD 217014 b	0.47	5755
12	HD 2638 b	0.48	5564
13	WASP-13 b	0.49	6025
14	WASP-34 b	0.59	5771
15	HD 209458 b	0.69	5988
16	HAT-P-30 b	0.71	6177
17	WASP-76 b	0.92	6133
18	WASP-74 b	0.97	5727
19	HAT-P-6 b	1.06	6442
20	HD189733 b	1.14	5374
21	WASP-82 b	1.24	6257
22	KELT-7 b	1.29	6460
23	HD 149143 b	1.33	6067
24	KELT-3 b	1.42	6404
25	KELT-2A b	1.49	6164
26	HD86081 b	1.50	6015
27	HAT-P-7 b	1.74	6270
28	HD 118203 b	2.14	5847
29	HAT-P-14 b	2.20	6490
30	WASP-38 b	2.71	6178
31	HD17156 b	3.20	5985
32	KELT-6 c	3.71	6176
33	HD 75732 d	3.86	5548
34	HD 115383 b	4.00	5891
35	HD 120136 b	5.84	6210
36	WASP-14 b	7.34	6195
37	HAT-P-2 b	8.74	6439
38	XO-3 b	11.79	6281